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MATHEMATICS 2 unit

Time allowed: three hours (*Plus 5 minutes reading time*)

Directions to candidates

- * Attempt ALL questions.
- * ALL questions are of equal value.
- * All necessary working should be shown in every question.
Marks will be deducted for careless or badly arranged work.
- * A standard integral sheet will be provided.
- * Board approved calculators may be used.
- * Each question attempted is to be returned on a separate sheet of paper clearly marked question 1, question 2, etc. at the top of the page.



QUESTION 1 *Start a new page.*

- a Calculate $\frac{11.7}{\sqrt{3 \cdot 9 \times \pi}}$ correct to two decimal places.
- b Factorise : $3x^2 - 11x + 6$
- c The distance of the Earth from the sun is 149 492 000 km.
Express this in scientific notation correct to two significant figures.
- d Solve : $\frac{2x-5}{4} = 7-x$
- e The time taken for one oscillation of a pendulum is given by $T = 2\pi\sqrt{\frac{L}{g}}$
where L is the length of the pendulum (in metres) and $g = 10 \text{ ms}^{-2}$ acceleration due to gravity.
Find the length of pendulum which takes 1.4 sec for one oscillation.

QUESTION 2 *Start a new page.*

- a
- On a number plane, mark the origin O and the points A(1, 3) and B(4, 2)
 - Find the gradients m_1 of OA and m_2 of AB.
 - Show that OA is perpendicular to AB.
 - Show that OA = AB
 - Find the midpoint D of the interval OB.
 - Find the coordinates of the point C such that D is the midpoint of AC.
 - What shape best describes the figure OABC ?
- b Thirty people are waiting in a queue for tickets. There are 12 males and 18 females.
If three people are chosen at random what is the probability that at least one of them is a male ?

QUESTION 3 *Start a new page.*

- a Differentiate :
- $2x^3 - 5x + \frac{3}{4x^3} - 7$
 - $5(3x^2 - x)^7$
 - $x \sin 2x$
 - $\frac{x}{1+x}$
- b Find the exact value of :
- $\int_0^1 e^{3x} dx$
 - $\int_0^5 \sqrt{3x+1} . dx$

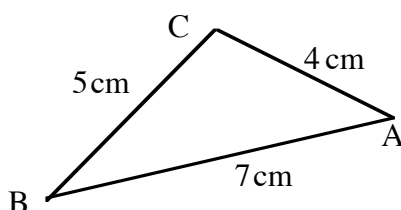


QUESTION 4 *Start a new page*

a If $y = \log_e (2 + x^2)$ find the value of $\frac{dy}{dx}$ when $x = 2$

b Find the exact value of $\cot 330^\circ \cdot \sec 240^\circ$

c Using the cosine rule ,
find the largest angle of ΔABC



d The second term of an arithmetic series 23 and the eighth term is 41

Find : i The first term and the common difference.

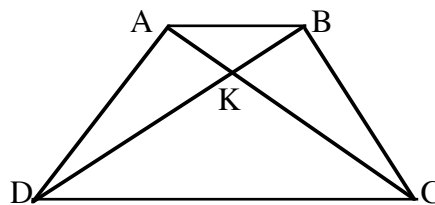
ii The sum of the first 50 terms.

QUESTION 5 *Start a new page.*

a Michael invests \$200 at the beginning of each month for 12 consecutive months.
If each monthly entry draws interest at the rate of 5% per annum,
find the total amount accumulated at the end of the twelve months.

b ABCD is a quadrilateral
with $AD = BC$ and diagonal $AC =$ diagonal BD

K is the point of intersection of the diagonals



i Copy the diagram onto your page.

ii Prove that triangle ABD is congruent to triangle ABC.

iii Prove that $\Delta AKD \cong \Delta BKC$ and hence that $KC = KD$

iv Name an angle equal to angle ADC (give a reason).

v Prove that AB is parallel to DC.



QUESTION 6 *Start a new page.*

- a** The perimeter of the figure shown is $(48 + 2x)$ metres.

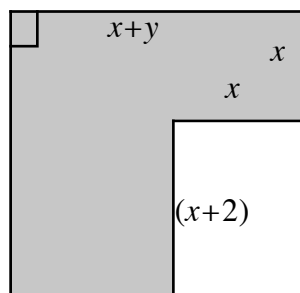


Figure not to scale.

All angles are right angles. All lengths are in metres.

- i** Express y as a function of x .

- ii** Show that the area of the figure is given by

$$A = x^2 + (2x + 2)(22 - 2x)$$

- iii** Find the values of x and y which will make the area a maximum.

- b** If α and β are the roots of $2x^2 + 3x - 4$ find the value of : **i** $\alpha + \beta$ **ii** $\alpha^2 + \beta^2$

- c** For what values of k will the expression $x^2 - kx + 1$ be positive definite ?

QUESTION 7 *Start a new page.*

- a** Let A and B be the fixed points $(-1, 0)$ and $(2, 0)$, and let P be the variable point (x, y)

- i** Write down expressions for PA and PB in terms of x and y .

- ii** Suppose that P moves so that $PA = 2PB$
Deduce that P moves on a circle.

- iii** Find the centre and radius of this circle.

- b** Eric invests \$50 000 in an account which earns 8% interest, compounded annually. He intends to withdraw \$M at the end of each year, immediately after the interest has been paid. He wishes to be able to do this for exactly 20 years, so that the account will then be empty.

- i** How much money is in the account immediately after he has made his first withdrawal ?

- ii** Write an expression in terms of M for the amount of money in the account, immediately after his 20th withdrawal.

- iii** Calculate the value of M which leaves his account empty after the 20th withdrawal.

- iv** Suppose Eric wished to be able to withdraw \$8000 per year for the 20 years. By using your calculator alone, estimate to the nearest per cent, the interest rate he would then need to earn on his account.

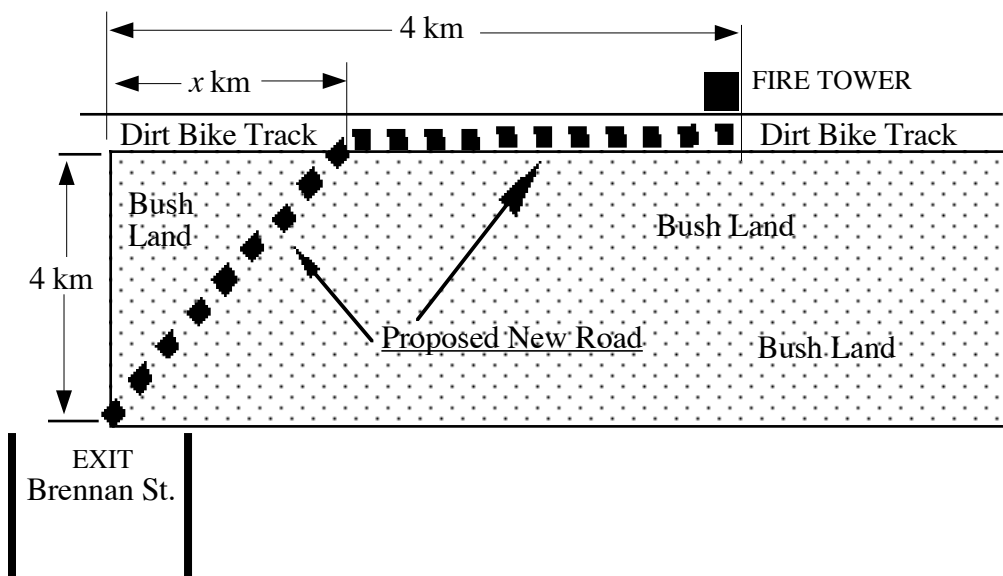


QUESTION 8 (start a new page)

Boomtown Council was asked to construct an all-weather vehicle access road from the end of Brennan Street to a fire tower in Jackson reserve.

The tower was adjacent to an existing dirt bike track running through the reserve.

The relative locations of the Brennan Street exit, the bush track and the fire tower are shown on the drawn diagram :



The proposed path is a straight line through the bush to the existing dirt bike track and then follow this track to the fire tower.(as shown above).

Cost /m of construction through the bush is \$50 , and along the dirt bike track the cost is \$30 /m.

Find the minimum cost of the proposed vehicle access road.

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QUESTION 9 (start a new page)

a Evaluate the following integrals correct to 2 decimal places :

i $\int_1^4 \left(t^3 - \frac{2}{t^3} \right) .dt$ ii $\int_0^2 e^{2x} .dx$

b The table below gives the values of $f(t)$ for $1 \leq t \leq 3$



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| | | | | | |
|--------|-----|-----|-----|-----|-----|
| t | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
| $f(t)$ | 0.0 | 1.8 | 4.2 | 2.8 | 2.0 |

Use Simpson's Rule with 5 function values
to evaluate : $\int_1^3 f(t) dt$ correct to 1 decimal place

- c Differentiate $x \cos x$ and hence or otherwise evaluate $\int_0^{\pi} x \sin x dx$ correct to 1 decimal place

QUESTION 10 (start a new page)

- a Thefenn brews his own beer in a barrel which is kept at a constant temperature. From past experience, he knows that the amount of sugar, M kg, that is present in the brew after t minutes, satisfies the equation : $M = M_0 \cdot e^{-kt}$ where k is a constant.
- i He places 15 kg of sugar in the barrel for his brew.
After 10 minutes he finds that only 5 kg of his sugar is present in the barrel.
Find the value of k correct to two decimal places.
- ii How long will it take before 1 kg of this sugar is present in the barrel ?
- b When Jane started work at age 17 years she decided to start saving for an overseas trip to be taken in four years time. She has decided that to pay for the trip she will need \$20000. Jane then commits herself to pay the same amount \$ P at the beginning of each month starting immediately. She will take out her investment at the end of the 48th month. If Jane's investment earns 0.5% per month and interest is compounded monthly, find the value of \$ P which will earn Jane a return of \$20000.