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Year 12 2 Unit Advanced

Test #3 Answers

Trial HSC

QUESTION 1

a $\neq 2\cdot 81$

b $x<2$

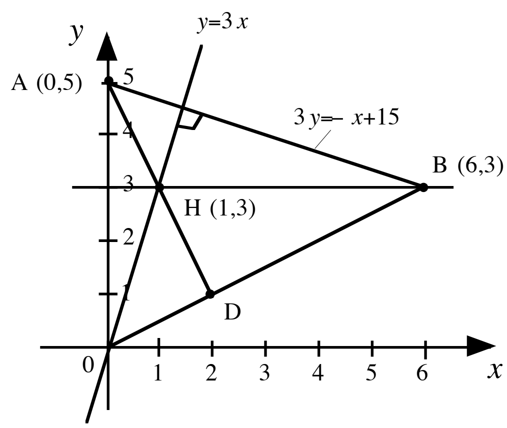
c i $u=\frac{\sqrt{3}-1}{2\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$
$=\frac{3-\sqrt{3}}{6}$

$v=\frac{1}{3-\sqrt{3}}\times\frac{3+\sqrt{3}}{3+\sqrt{3}}$
$=\frac{3+\sqrt{3}}{6}$

ii $uv=\frac{3-\sqrt{3}}{6}\times\frac{3+\sqrt{3}}{6}$
$=\frac{9-3}{36}$
$=\frac{1}{6}$ rational

d Sum Mult of $3<200=\frac{66}{2}(3+198)$
$=6633$

QUESTION 2



a Slope $AB=-\frac{1}{3}$
$\therefore$ Slope $\perp$ AB is 3
$\therefore$ Line $\perp$ AB through O(0,0) is $y=3x$

b Line through $B\perp OA$ is $y=3$

c H: $y=3x, \cap\ y=3\ \therefore H(1,3)$

d Slope OB $=\frac{3}{6}$; Slope AH $=-\frac{2}{1}$
$=\frac{1}{2}$ $\qquad =-2$

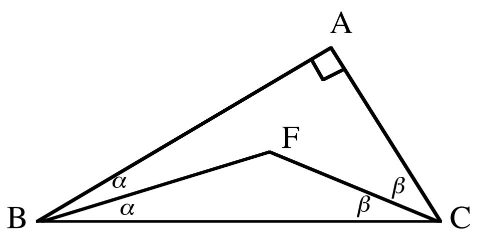
Slope OB $\times$ Slope H $=\frac{1}{2}\times -2=-1$; AH$\perp$OB

e AH is $y=-2x+5\ \cap$ OB: $y=\frac{1}{2}x$ when $\frac{1}{2}x=-2x+5$ $\quad x=2,\ y=1\ \therefore D(2,1)$

f Area $\Delta OHB=\frac{1}{2}$ base $\times$ height $=\frac{1}{2}BH\times 3$
$=7\frac{1}{2}u^2$

QUESTION 3

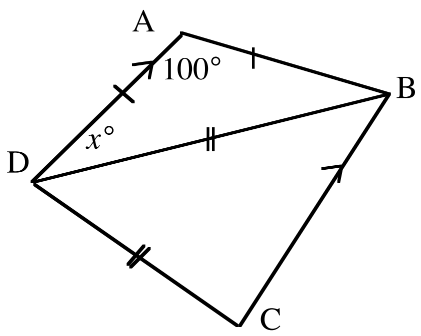
a



$\angle ABF = \angle FBC \ (= \alpha \text{ say})$ data
$\angle ACF = \angle FCB \ (= \beta \text{ say})$ data
$\angle BAC = 90°$ data
$\therefore$ Angle Sum ΔABC is
$90° + 2\alpha + 2\beta = 180°$
$\therefore \alpha + \beta = 45°$

Angle Sum ΔBFC is $\alpha + \beta + \angle BFC = 180°$ $(\alpha + \beta = 45°)$
$\therefore \angle BFC = 135°$

b



$\angle ADB = \angle ABD = x°$ (base $\angle$'s of isos Δ)
$\therefore 2x + 100 = 180°$ ($\Sigma\angle$ of ΔABD)
$\therefore x = 40°$
$\angle ADB = \angle DBC = 40°$ (Alt Δ AD//BC)
$\therefore \angle DCB = \angle DBC = 40°$ (base $\angle$ of isos Δ)
$\therefore \angle BDC = 180 - (40 + 40)°$ ($\Sigma\angle$ of Δ)
$= 100°$

c Sum internal $\angle$'s of hexagon $= (2 \times 6 - 4)$ rt $\angle$'s
$= 720°$
$\therefore$ Size of each $\angle$ of regular hexagon $= 720 \div 6$
$= 120°$

QUESTION 4

a $(k-1)(k+4) < 0$; equal points $k = -4$ or 1
$\therefore$ true $k = 0$ $\therefore$ $-4 < k < 1$

b $kx^2 + (k+3)x + 4$ is positive definite if (i) $k > 0$ (ii) $\Delta < 0$
$\Delta = (k+3)^2 - 16k < 0$ ie $k^2 - 10k + 9 < 0$
$\therefore 1 < k < 9$
$\therefore kx^2 + (k+3)x + 4$ is pos definite when $1 < k < 9$
The equation is neg definite if (i) $k < 0$ and (ii) $\Delta < 0$ but when $k < 0, \Delta > 0$ $\therefore$ never neg def.

c Roots of $2x^2 - 15x + C = 0$ are $\alpha,\ 4\beta$; $\therefore\ 5\alpha = \frac{15}{2}$

ie $\alpha = \frac{3}{2}$ $\therefore$ Roots $\frac{3}{2}$, 6

Prod of roots is $\frac{C}{2} = 9\ \therefore\ C = 18$

QUESTION 5

a i $-\frac{2}{x\sqrt{x}}$ **ii** $\frac{4 - 6x - x^2}{(4 + x^2)^2}$

b i Slope $\underset{\sim}{t}$ is $\frac{dy}{dx} = 2x - 3$ when $x = 2,\ m = 1$ $\therefore$ Equat of $\underset{\sim}{t}$ is $y = x - 4$

ii Circle $x^2 + y^2 = 8$ has centre (0,0) and radius $= \sqrt{8}$

Perp dist of (0,0) to $y = x - 4$ is $d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right| = \frac{4}{\sqrt{2}} = \sqrt{8}$

Since $\perp$ from centre to line = radius then line is tangent.

QUESTION 6

a $f(x) = 3 - 3x^2 - x^3$ has stationary points when

$f'(x) = 0$ ie $6x + 3x^2 = 0$

$x = 0$ or -2 $\therefore$ pts (0,3); (-2,-1)

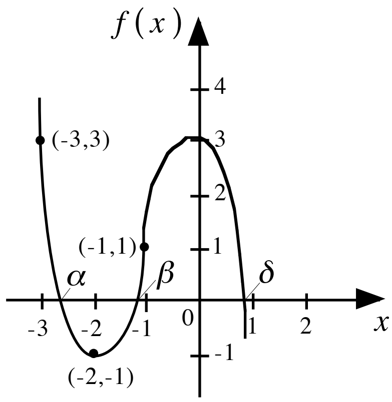
b $f''(x) = -6 - 6x$; $f''(0) = -6\ \therefore\ (0,3)$ max

$f''(-2) = 6\ \therefore\ (-2,-1)$ min

c Inflexion $f''(x) = 0\ \therefore$ Pt $(-1,1)$ Note $f'(1) = -1$

$f(-3) = 3$

d



e Roots α, β, δ as shown $-3 < \alpha < -2$, $-2 < \beta < -1$, $0 < \delta < 1$

QUESTION 7

a **ii** $\cot\alpha = \dfrac{a+b}{a}$; $\cot\beta = \dfrac{b}{a}$

$\cot\alpha - \cot\beta = \dfrac{a+b}{a} - \dfrac{b}{a} = 1$

iii $\beta = 57°$, $\cot\alpha = 1 + \cot 57°$

$\alpha = 31°$

b $\cos\angle BAC = \dfrac{3^2 + 7^2 - 5^2}{2 \times 3 \times 7} = \dfrac{11}{14}$

$\cos\angle ACD = \dfrac{7^2 + 8^2 - 5^2}{2 \times 7 \times 8} = \dfrac{11}{14}$

$\therefore \angle BAC = \angle ACD$ but these are alt $\angle$'s $\therefore$ BA//CD