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Year 12 2 Unit Advanced Worked Solutions Test #1

QUESTION 1

a $3{\cdot}34$

b $(3x-2)(x-3)$

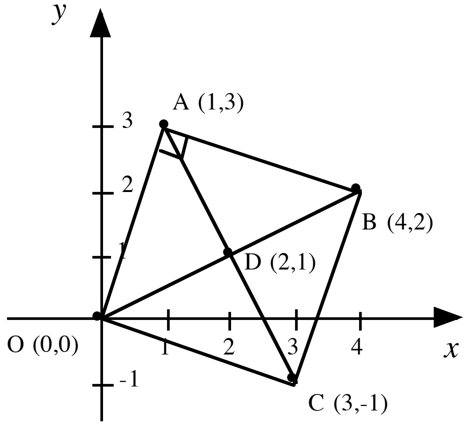
c $1{\cdot}5\times10^{8}$ (2 significant figures)

d
$$\frac{2x-5}{4}=7-x$$
$$2x-5=28-4x$$
$$6x=33$$
$$x=5\frac{1}{2}$$

e
$$T=2\pi\sqrt{\frac{L}{g}}$$
$$L=\frac{gT^{2}}{4\pi^{2}}\qquad g=10,\ T=1{\cdot}4$$
$$\approx 0{\cdot}496\,\text{m}$$

QUESTION 2

a i



ii $m_1=\text{Slope OA}=3$

$m_2=\text{Slope OB}=\frac{1}{2}$

iii Slope AB $m_3=-\frac{1}{3}$

since $m_1m_3=3\times-\frac{1}{3}=-1$

then $OA\perp AB$

iv $OA=\sqrt{1^2+3^2}$

$=10$

$AB=\sqrt{1^2+3^2}$

$=\sqrt{10}$

$\therefore$ OA = AB

v Mid-pt of OB is $D\equiv(2,1)$

vi $C=(3,-1)$

vii OABC is a square

b 12 males, 18 females. Pr (at least 1 male) = 1− Pr (all females)

$\therefore$ Pr (at least 1 male) $=1-\frac{18}{30}\times\frac{17}{29}\times\frac{16}{28}$

$=\frac{811}{1015}\ (\approx 0{\cdot}7990)$

2U Yr 12 Test 05 Ans

QUESTION 3

a i
$$y=2x^{3}-5x+\frac{3}{4x^{3}}-7$$
$$y'=6x^{2}-5-\frac{9}{4x^{4}}$$

ii
$$y=5\left(3x^{2}-x\right)^{7}$$
$$y'=35\left(3x^{2}-x\right)^{6}(6x-1)$$

iii $y = x\sin 2x$

$\frac{dy}{dx} = \sin 2x + 2x\cos 2x$

iv $y = \frac{x}{1+x}$

$\frac{dy}{dx} = \frac{1}{(1+x)^2}$

b i $\int_0^1 e^{3x}\,dx = \frac{1}{3}e^{3x}\Big|_0^1$

$= \frac{1}{3}\left(e^3 - 1\right)$

ii $\int_0^5 \sqrt{3x+1}\,.dx = \frac{2}{9}(3x+1)^{3/2}\Big|_0^5$

$= \frac{2}{9}(64-1)$

$= 14$

QUESTION 4

a $y = \ln\left(2+x^2\right)$ when $x = 2$, $\frac{dy}{dx} = \frac{2}{3}$

$\frac{dy}{dx} = \frac{2x}{2+x^2}$

b $\cot 330°.\sec 240° = -\sqrt{3}\times -2$

$= 2\sqrt{3}$

c $\cos C = \frac{4^2+5^2-7^2}{2\times 4\times 5}$ $\therefore$ Largest angle $= 101°32'$

d $T_8 = a + 7d = 41$ $\therefore$ $6d = 18$

i $T_2 = a + d = 23$ $d = 3$ $a = 20$

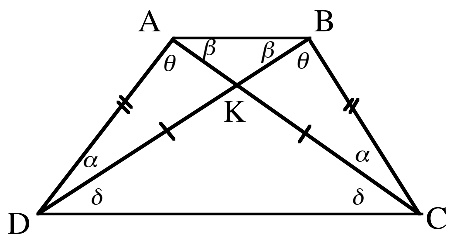
ii $S_{50} = \frac{50}{2}(40 + 49\times 3) = 4675$

QUESTION 5

a Michael investment $= \dfrac{\$200\left(1\frac{1}{240}\right).\left[\left(1\frac{1}{240}\right)^{12}-1\right]}{\frac{1}{240}}$

$= \$2466\cdot00$

b i



ii In Δ's ABD, ABC
AD = BC (data)
BC = AC (data)
AB = AB common side
$\therefore$ ΔABD≡ΔABC (S.S.S.)

$\therefore$ $\angle ADB = \angle BCA = \alpha$ (corresp angles in cong Δ's) and $\angle BAC = \angle ABD = \beta$
$\therefore$ ΔAKB is isosceles with AK = BK
$\therefore$ In ΔAKD, BKC
AK = BK proved ; $\angle ADK = \angle BCA = \alpha$ proved
$\angle AKD = \angle BKC$ (vert opp) $\therefore$ ΔAKD≡ΔBKC (A.A.S.)

iii $\therefore$ KC = KD (corresp sides in cong Δ's)

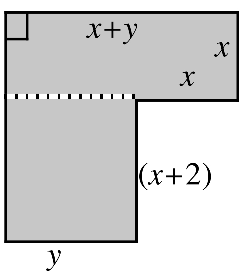
iv $\angle ADC = \angle BCD$ (ΔADC ≡ ΔBCD S.S.S.)

v Note: ΔABK///ΔDKC (isos with vert ∠'s equal)
$\therefore$ $\angle BAC = \angle BDC$ but these are alt $\therefore$ AB//DC

QUESTION 6

a i Per = $2(x+y)+2(2x+2)=48+2x$
$\therefore\ y+3x=22+x$ or $y=22-2x$

ii Shaded Area $A(x)=x(x+y)+y(x+2)$



$= x^2+2xy+2y$
$= x^2+(2x+2)(22-2x)$
$= -3x^2+40x+44$

$A(x)$ is a max when $A'(x)=0$ & $A''(x)<0$
$\therefore\ -6x+40=0$ $\quad A''(x)=-6<0$ $\therefore$ max
$x=6\frac{2}{3}$

$\therefore$ Max Area when $x=6\frac{2}{3}$m, $y=8\frac{2}{3}$

QUESTION 6 (cont)

b $2x^2+3x-4=0$ has roots α,β

i $\alpha+\beta=-\frac{3}{2},\ \alpha\beta=-2$ **ii** $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=6\frac{1}{4}$

c $x^2-kx+1=0$ Pos def $\Delta<0$ ie $-2<k<2$

QUESTION 7

a **i** $PA=\sqrt{(x+1)^2+y^2}$ $PB=\sqrt{(x-2)^2+y^2}$, $PA=2PB$

ii $PA^2=4PB^2$ $(x+1)^2+y^2=4(x-2)^2+4y^2$

$y^2+x^2-6x+5=0$ circle

iii $(x-3)^2+y^2=4$ circle $(3,0)$; $r=2$

b **i** Amt after Yr 1 $=\$50000(1\cdot08)-M$

Amt after Yr 2 $=[\$50000(1\cdot08)-M]1\cdot08-M$

$=\$50000(1\cdot08)^2-M(1+(1\cdot08))$

ii Amt after 20th withdrawal

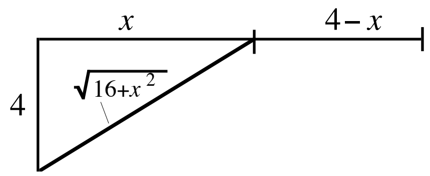
$0=\$50000(1\cdot08)^{20}-M\left(1+(1\cdot08)+\ldots+(1\cdot08)^{19}\right)$

iii $M=\frac{50000(1\cdot08)^{20}}{(1\cdot08)^{20}-1}\times0\cdot08$

$\approx\$5092\cdot61$

iv Interest of 15% gives \$8000/Yr

QUESTION 8



Cost $C(x) = \left[\sqrt{16+x^2}\times\$50+\$30(4-x)\right]\times\$1000$

Cost is a Min when $C'(x)=0$ and $C''(x)>0$

$$C'(x)=\left[\frac{50x}{\sqrt{16+x^2}}-30\right]\$1000;$$

$$C''(x)=\frac{\sqrt{16+x^2}-\frac{x^2}{\sqrt{16+x^2}}}{16+x^2}=\frac{16}{(16+x^2)\sqrt{16+x^2}}>0$$

$C'(x)=0$ when $x=\pm 3$

Min Cost = \$280,000

ANSWER 9

a **i** $\int_1^4\left(t^3-\frac{2}{t^3}\right).dt=\frac{t^4}{4}+\frac{1}{t^2}\Big|_1^4$

$\approx 62\cdot 81\left(62\frac{13}{16}\right)$

ii $\int_0^2 e^{2x}.dx=\frac{1}{2}e^{2x}\Big|_0^2$

$\approx 26\cdot 80$

b Area $\int_1^3 f(t)dt\approx\frac{1}{6}[1\times 0+4\times 1\cdot 8+2\times 4\cdot 2+4\times 2\cdot 8+1\times 2\cdot 0]$

$\approx\frac{1}{6}\times 28\cdot 8$

$\approx 4\cdot 8$

c $y=x\cos x\ \therefore\ \frac{dy}{dx}=\cos x-x\sin x$

$\therefore\int_0^\pi(\cos x-x\sin x)dx=x\cos x\big|_0^\pi=-\pi$

$$\therefore \int_0^{\pi} x\sin x\,dx = \int_0^{\pi} \cos x\,dx + \pi$$
$$\approx 3\cdot 1$$

ANSWER 10

a **i** $M = M_0 e^{-kt}$; $M_0 = 15$ and when $t = 10, M = 5$

$\therefore 5 = 15e^{-kt}$ $t = 10$

$e^{10k} = 3$

$k = \frac{1}{10}\ln 3$

$\approx 0\cdot 11$

ii $M = 1$ when $15e^{-kt} = 1$ $\therefore e^{kt} = 15$

$t = \frac{1}{k}\ln 15$

$\approx 24\cdot 65$minutes

b Let amount/month = \$P $\therefore \frac{\$P(1\cdot 005)\left[(1\cdot 005)^{48} - 1\right]}{1\cdot 005 - 1} = \$20,000$

ie $\$P = \frac{\$20000 \times \cdot 005}{1\cdot 005\left[(1\cdot 005)^{48} - 1\right]}$

$\approx \$367\cdot 86$