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Year 12 2 Unit Extension 2

Test #11

TRIAL HSC

MATHEMATICS Year 12 Extension 2

Time allowed: three hours *(Plus 5 minutes reading time)*

Directions to candidates :

* Attempt ALL questions.

* ALL questions are of equal value.

* All necessary working should be shown in every question.
  Marks will be deducted for careless or badly arranged work.

* Board approved calculators may be used.

* Each question attempted is to be returned on a separate sheet of paper
  clearly marked question 1, question 2, etc. at the top of the page.

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Question 1 *start new page*

a Evaluate : **i** $\int_1^e x^2 \ln x \, .dx$ **ii** $\int_2^3 \frac{2x-1}{x^2+x-2} .dx$

b Show that $\int_0^{\pi/4} \left(\tan^3 x + \tan x\right).dx = \frac{1}{2}$ and hence evaluate $\int_0^{\pi/4} \tan^3 x \, .dx$

c Prove that if $u_n = \int_0^a x^n e^{-x} \, .dx$ then $u_n - n \, .u_{n-1} = -e^{-a} \, .a^n$

Question 2 *start new page*

a Sketch the curve $y = \frac{2x}{x^2-1}$ $(x \neq \pm 1)$ showing the equations of all the asymptotes.

b If α and β are the roots of $x^2 + px + q = 0$,
show that p and q are the roots of $x^2 + (\alpha + \beta - \alpha\beta)x - \alpha\beta(\alpha + \beta) = 0$.
Find the non-zero values of p and q if the roots of the second equation are α^2 and β^2 .

Question 3 *start new page*

The hyperbola H has cartesian (x, y) equation $12x^2 - 4y^2 = 27$

Write down its eccentricity, the coordinates of its foci S and S′,
the equation of each directrix and the equation of each asymptote.

Sketch the curve and indicate on your diagram the foci , directrices , and asymptotes.

P is an arbitary point on H :

i Derive the equation of the tangent at P.

ii This tangent meets a directrix at T. Show that PT subtends a right angle at the focus.

Question 4 *start new page*

a State the modulus and argument of the complex number $w = \sqrt{3} + i$

Mark on a half page sized Argand diagram the following points :

A corresponding to w. B corresponding to w^3

C corresponding to w^2 D corresponding to $\sqrt{w}$

b Given that $z = x + iy$ and $w = \frac{z-1}{z}$ express w in the form $u + iv$ where u and v are real and show that if $|z| = 4$ then $|w - 1| = \frac{1}{4}$. Give a geometrical description of this situation.

c What is the locus in the Argand diagram of the point which represents the number z , where $z\bar{z} - 6(z + \bar{z}) = 45$

Question 5 *start new page*

a If $f(x)+f(a-x)=f(a)$ prove that $\int_0^a f(x).dx = \frac{1}{2}a.f(a)$

b A rectangle is inscribed in a semi-circle of radius a. Find the maximum area of the rectangle.

c The area enclosed between the curve $y^2 = x^2 - a^2$ and the chord $x = 2a$,
is rotated about the y axis.
Prove that the volume of the solid generated is $4\pi a^3\sqrt{3}$ units of volume.

Question 6 *start new page*

A particle is projected with a speed Vm/s at an angle α with the horizontal,
up a plane inclined at an angle θ with the horizontal.

i Show that the range R of the particle along the plane is given by $R = \frac{2V^2\cos\alpha.\sin(\alpha-\theta)}{g\cos^2\theta}$

ii and that the maximum range (for small θ), is $R = \frac{V^2}{g(1+\sin\theta)}$

Question 7 *start new page*

a The polynomial $P(x)$ is given by $P(x)=x^5-5cx+1$ where c is a real number.

i Consider the turning points ;
prove that if $c < 0$, then P(x) has just one real root which is negative.

ii Prove that P(x) has three real roots if and only if $c > \left(\frac{1}{4}\right)^{\frac{4}{5}}$

b A committee of 5 is to be chosen from ten persons , of whom 6 are men and 4 are women.
Assume all choices are equally likely.
What is the probabilty that the women are a majority of the committee ?

Question 8 *start new page*

Find the complete factorisation of $p(z)=z^6-1$: **i** over the real field R
ii over the complex field C

iii Write down the values of z for which $p(z) = 0$ in both $x + iy$ form and modulus argument form.
For each such root α evaluate $1+\alpha^2+\alpha^4$