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**Year 12 2 Unit Extension 1**

**Test #7 Answers**

**QUESTION 1**

**a** $\frac{1}{4}e^{2x}+c$

**b** $y=\cos x^2$

$y'=-2x\sin x^2$

$\frac{d^2y}{dx^2}=-2\sin\left(x^2\right)-4x^2\cos\left(x^2\right)$

**c** $\int_1^4 t\sqrt{t}.dt=\frac{2}{5}t^2\sqrt{t}\Big|_1^4$

$=\frac{2}{5}(32-1)$

$=12\frac{2}{5}$

**d** $\int_2^{20}\log_{10}x.dx\approx\frac{18(f(2)+4f(11)+f(20))}{6}$

$\approx 17\cdot 303 \quad (3\text{dp})$

**QUESTION 2**

**a** $\sum_{k=3}^{k=n}(4k-2) \quad S_m=\frac{m}{2}[14+4m+2]=2040$ where $m=n-2$

$2m^2+8m=2040$

$m^2+4m-1020=0$

$(m+34)(m-30)=0$

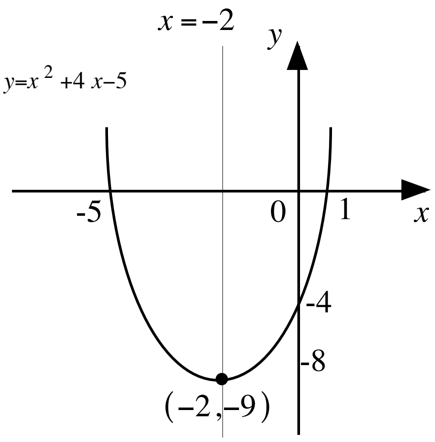
$m=30$

so $m=30$

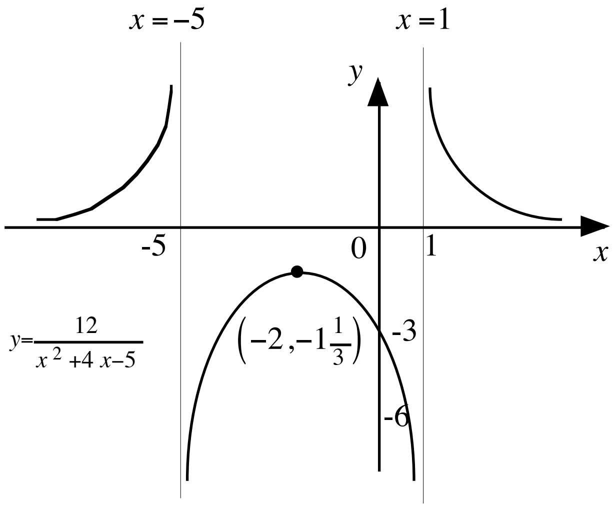
(Note: $n\geq 3$)

$\therefore \quad n=32 \quad$ ie $\sum_{k=3}^{32}(4k-3)=2040$

**b i**




**ii**




Max (Reciprocal of Min) at $\left(-2,-1\frac{1}{3}\right)$

Asymptotes $x=-5$ and $x=1$ and $y=0$

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**QUESTION 3**

**a** $T_n = 3n - 1$

$T_2 n = 6n - 1$

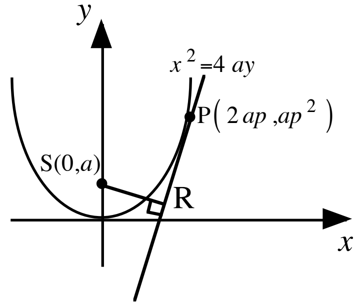
$T_1 = 2$

$S_{2n} = \frac{2n}{2}[T_1 + T_{2n}]$

$= n[+2 + 6n - 1]$

$= 6n^2 + n$

**b**




$P(2ap, ap^2)$ represents a point on the parabola $x^2 = 4ay$ for all values of P.

Slope of tangent at P is $\frac{dy}{dx} = \frac{y'(p)}{x'(p)} = \frac{2ap}{2a} = p$

Equat of tangent at P is $y = px - ap^2$

Let P $= \frac{1}{\lambda}$ then $\underline{t}$ is $y = \frac{x}{\lambda} - \frac{\alpha}{\lambda^2} \times \lambda$

ie $\lambda y + \frac{a}{\lambda} = x$ is $a$

Tangent for all $\lambda . (\lambda \neq 0)$

Slope of SR $= -\frac{1}{p} = -\lambda$

$\therefore$ Equat of SR is $\lambda x + y = a$ (passes through $(0,a)$)

SR intersects tangent at P when

$$\left.\begin{aligned} x &= \lambda y + \frac{a}{\lambda} \\ \lambda x + y &= a \end{aligned}\right\} \text{ simultaneous}$$

$\lambda\left(\lambda y + \frac{a}{\lambda}\right) + y = a$

$\lambda^2 y + y + a - a$

$y\left(\lambda^2 + 1\right) = 0$

Note $\left(\lambda^2 + 1\right) \neq 0$ $\quad \therefore \; y = 0$

$\therefore$ Locus of R is $y = 0$ (ie the $x$ axis)

**c** Step 1 $\quad n = 1 \quad$ LHS $= \frac{1}{1 \times 2} = \frac{1}{2}$ $\quad$ RHS $= 1 - \frac{1}{2}$ $\quad \therefore$ Step 1 true

$= \frac{1}{2}$

Step 2 $\quad$ assume true for $n = k$

assume $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \ldots \frac{1}{k(k+1)} = 1 - \frac{1}{k+1}$

Step 3 $\quad$ Prove true for $n = k + 1$ prove

$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \ldots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} = 1 - \frac{1}{k+2}$

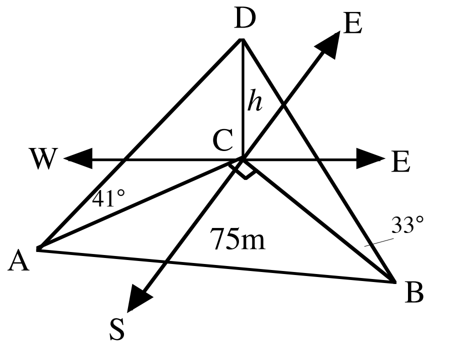
Proof: LHS $=1-\dfrac{1}{k+1}+\dfrac{1}{(k+1)(k+2)}$ by step 2

$=1-\dfrac{[(k+2)-1]}{(k+1)(k+2)}$

$=1-\dfrac{1}{k+2}$ Q.E.D. etc

## QUESTION 4

**a**




Tower CD

$AC = h\cot 41° = h\tan 49°$

$BC = h\tan 57°$

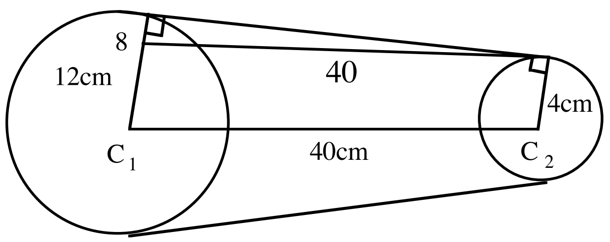
$AC^2 + BC^2 = 75^2$

$h^2\tan^2 49° + h^2\tan^2 57° = 75^2$

$h^2 = \dfrac{75^2}{\tan^2 49° + \tan^2 57°}$

$\therefore\ h = 39{\cdot}0\text{m}\,(1\text{dp})$

**b**

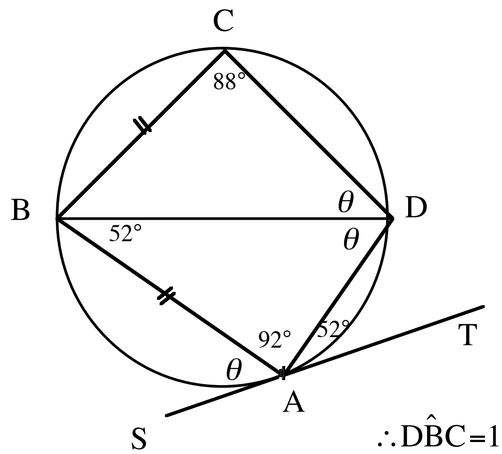



$\sin\theta = \dfrac{8}{40} = 0{\cdot}2$

$\theta = 11°32'$

Angle between two straight lengths of chain is $2\theta = 23°4'$

c




$A\hat{B}D = D\hat{A}T = 52°$ (Angle between chord and tangent equals angle in alt seg.)

Let $B\hat{A}S = \theta$

$\therefore\ B\hat{D}A = B\hat{A}S = \theta$ ( " )

Also $B\hat{D}C = B\hat{D}A = \theta$ (Subt by equal arcs)

$B\hat{A}D = 180 - B\hat{C}D$ (Opp $\angle$'s of cyclic Q)

$\therefore\ B\hat{A}D = 92°$

$\therefore\ B\hat{A}S = \theta = 36°$ ($\sum\angle$'s of ST line SAT)

$\therefore D\hat{B}C = 180° - (88 + \theta) = 56°$ ($\sum$ Angles of $\Delta$BCD)

# Year 12 2 Unit Extension 1

# Test #7 Answers

## QUESTION 5

**a** $N = N_0 e^{-kt}$, $N_0 = 1200$ $\quad t = 0$ in 1960

$t = 10 \quad 1000 = 1200e^{-10k}$

$e^{10k} = \frac{6}{5}$

$k = 0 \cdot 018$

$N = 800$ then

$800 = 1200e^{-kt}$

$e^{kt} = \frac{3}{2}$

$t = \frac{1}{k}\ln\left(\frac{3}{2}\right)$

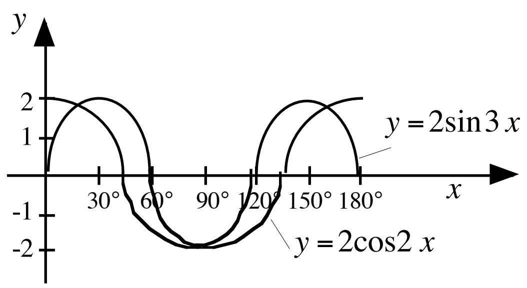
$t \approx 22 \cdot 2$ Yrs $\therefore$ Pop reach 800 in 1982

**b** Pr (sell a pie to a customer next day)
= Pr (Rain & Sell pie) or Pr (Fine & Sell pie)
$= 0 \cdot 8 \times 0 \cdot 6 + 0 \cdot 2 \times 0 \cdot 3$
$= 0 \cdot 54$
$\therefore$ Customers expect to buy $50 \times 0 \cdot 54 = 27$ pies next day.

**c** Roots of $ax^2 + bx + c = 0$ are imaginary
$\Leftrightarrow \quad b^2 - 4ac < 0$ note $\frac{a}{b} = \frac{b}{c}$ ie $b^2 = ac$
$\therefore b^2 - 4ac = b^2 - 4b^2 = -3b^2 < 0$
$\therefore$ Roots of $ax^2 + bx + c = 0$ are imaginary if *a*, *b*, *c* are in proportion.

## QUESTION 6

**a**




$\cos x = \sin 3x$ when $x \approx 20°, 90°$ or $170°$

**b** Aim : To show $\frac{\cos A.\sec A}{\operatorname{cosec} A} \equiv \frac{\tan A}{\sqrt{1+\tan^2 A}}$

$\text{LHS} = \frac{1}{\operatorname{cosec} A}$ $\quad (\cos A \sec A = 1)$

$= \sin A$

$\text{RHS} = \frac{\tan A}{\sqrt{\sec^2 A}}$

$= \frac{\sin A}{\cos A \sec A}$

$= \sin A$

$\therefore \text{LHS} = \text{RHS} = \sin A$

QED

**c** $\frac{3x}{x^2-4} \geq 1 \quad x \neq -2 \text{ or } 2$

Solve equality first

$3x = x^2 - 4$

$x^2 - 3x - 4 = 0 \quad$ Test $x = 0 \quad 0 \geq 1 \quad$ False

$x = 4 \text{ or } -1$

$\therefore -2 < x \leq 1 \text{ or } 2 < x \leq 4$

## QUESTION 7

**a** $\int_0^4 \frac{3x-1}{\sqrt{2x+1}}.dx = \int_1^9 \frac{\left[3\left(\frac{u-1}{2}\right)-1\right]\frac{1}{2}du}{\sqrt{u}}$ $\quad u = 2x+1; \; x = \frac{u-1}{2}$

$du = 2\,dx$

$= \frac{1}{4}\int_1^9 \frac{(3u-3)-2}{\sqrt{u}}du$ $\quad x = 0, \; u = 1$

$x = 4, \; u = 9$

**b** **(i)** $\int_0^4 \frac{3x-1}{\sqrt{2x+1}}dx = \frac{1}{4}\int_1^9 \frac{3u-5}{\sqrt{u}}du$

$= \left[\int_1^9 \sqrt{u} - \frac{5}{4\sqrt{u}}du\right]$

$= \left[\frac{3}{4}\times\frac{2}{3}u\sqrt{u} - \frac{5}{2}\sqrt{u}\right]_1^9$

$= \frac{u\sqrt{u}}{2} - \frac{5}{2}\sqrt{u}\Big|_1^9$

$= \frac{1}{2}(27-1) - \frac{5}{2}(3-1)$

$= 13-5$

$= 8u^2$

**(ii)** $y = u^3 - 3u \qquad x = 2u^2 - 3$

Slope of tangent is $\frac{dy}{dx} = \frac{y'(u)}{x'(u)} = \frac{3u^2-3}{4u}$

$u=2$ Point (5,2) Slope $= \frac{9}{8}$

$\therefore$ Equation of tangent is $\quad 8y = 9x - 29$

$(5,2) \quad 16 = 45 - 29$