



**Master Coaching**  
**Ph:1800 TUITION**  
**www.mastercoaching.com**

**Year 12 2 Unit Extension 1**

**Test #7**

## **Ext 1**

Time allowed : two hours      (*Plus 5 minutes reading time*)

### Directions to candidates.

- \* Attempt ALL questions.
- \* ALL questions are of equal value.
- \* All necessary working should be shown in every question.  
Marks will be deducted for careless or badly arranged work.
- \* A standard integral sheet will be provided.
- \* Board approved calculators may be used.
- \* Each question attempted is to be returned on a separate sheet of paper clearly marked question 1, question 2, etc. at the top of the page.



**QUESTION 1** (Start a new page)

- a Find a primitive of  $\frac{1}{2}e^{2x}$
- b Find the second derivative of  $\cos(x^2)$
- c Evaluate  $\int_1^4 t\sqrt{t} \cdot dt$
- d Use Simpson's Rule with three points to approximately evaluate  $\int_2^{20} \log_{10} x \cdot dx$

**QUESTION 2** (Start a new page)

- a Given that  $\sum_{k=3}^{k=n} (4k - 2) = 2040$ , find the value of n
- b On separate axes, sketch the two curves **i**  $y = x^2 + 4x - 5$  and **ii**  $y = \frac{12}{x^2 + 4x - 5}$   
Show all turning points, asymptotes and points of inflexion.

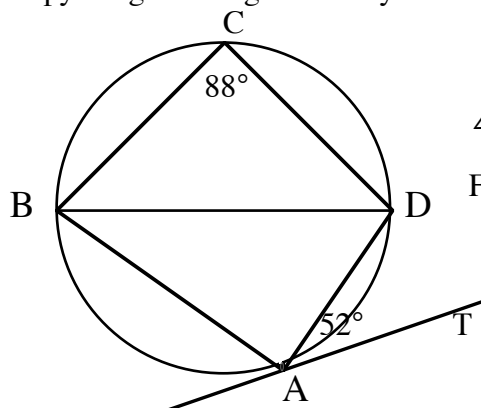
**QUESTION 3** (Start a new page)

- a For the series in which  $T_n = 3n - 1$  show that  $S_n = 6n^2 + n$
- b Prove that the line  $x = \lambda y + \frac{a}{\lambda}$  is a tangent to  $x^2 = 4ay$  for all values of  $\lambda$   
A perpendicular is drawn to this line from the focus.  
If the foot of this perpendicular is R, find the equation of the locus of R as  $\lambda$  varies.
- c Prove by mathematical induction that  $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{n(n+1)} = 1 - \frac{1}{n+1}$



**QUESTION 4** (Start a new page)

- a** From a point A, South West of a tower of height  $h$  m, the angle of elevation of the top of the tower is  $41^\circ$ . From point B, 75m from A, the angle of elevation of the top of the tower is  $33^\circ$ . Draw a diagram using the given information. Find the height of the tower, given that B is South East of the tower and the points A, B and the base of the tower all lie in a horizontal plane.
- b** A bicycle is driven by two sprockets, one 12cm radius at the front, and the other 4cm radius. If the centres are 40cm apart find the angle between the two straight lengths of chain.
- c** ABCD is a cyclic quadrilateral and AT is a tangent at point A. Copy the given diagram onto your answer paper.



$\angle BCD = 88^\circ$  ,  $\angle DAT = 52^\circ$  ,  $BC = AB$

Find  $\angle DBC$  giving reasons.

**QUESTION 5** (Start a new page)

- a** It was found that the population  $N$ , of a certain tribe was decreasing at a rate proportional to  $N$ . The population at the beginning of the study in 1960 was known to be 1,200 and 10 years later it was 1,000. When, i.e. in what year, will the population reach 800 ?
- b** A canteen has 50 regular customers. The probability that a customer orders a meat pie is 0.3 if it is a fine day and 0.6 on a wet day. The probability that it will rain the next day is 0.8. How many pies would the caterer expect to sell on the next day ?
- c** If  $a, b, c$  are in proportion ( i.e.  $\frac{a}{b} = \frac{b}{c}$  ) , show that the roots of  $ax^2 + bx + c = 0$  are imaginary.



**QUESTION 6**

*(Start a new page)*

- a** On the same axes sketch the curves  $y=2\cos 2x$  and  $y=2\sin 3x$  for  $0^\circ \leq x \leq 180^\circ$  and use the sketch to estimate the value(s) of  $x$  in the given domain for which  $\cos 2x = \sin 3x$
- b** Prove that  $\frac{\cos A \cdot \sec A}{\operatorname{cosec} A} \equiv \frac{\tan A}{\sqrt{1 + \tan^2 A}}$
- c** Solve  $\frac{3x}{x^2 - 4} \geq 1$

**QUESTION 7**

*(Start a new page)*

- a** Evaluate  $\int_0^4 \frac{3x-1}{\sqrt{2x+1}} \cdot dx$  using the substitution  $u=2x+1$
- b** Find the equation of the tangent to the curve  $y=u^3 - 3u$  and  $x=2u^2 - 3$  at the point where  $u=2$