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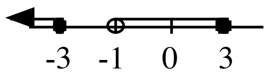
Year 12 2 Unit Extension 1

Test #8 Answers

QUESTION 1

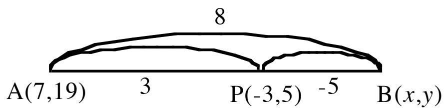
a $\frac{8}{x+1} \geq x-1;\ x=-1$ solve equality

$x=3$ or -3



$\therefore\ x \leq -3$ or $-1 < x \leq 3$

b

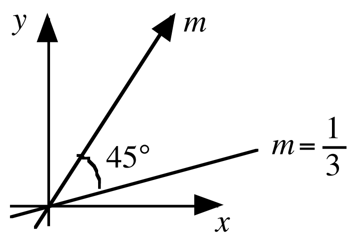


$x=\frac{8x-3+5x-7}{8+-5}$; $y=\frac{8\times 5+5x-19}{3}$

$=-19\frac{2}{3}$ $=-18\frac{1}{3}$

$\therefore\ B\left(-19\frac{2}{3},15\right)$

c



$\frac{m-\frac{1}{3}}{1+\frac{1}{3}m}=\tan 45°=1$

$\frac{3m-1}{m+3}=1$

$m=2$

d SYDNEY No of 4 letter words 2Y's $= {}^4C_2 \times \frac{4!}{2!} = 72$

No of 4 letter words 1Y $= {}^4C_3 \times 4! = 96$

No of 4 letter words 0Y $= 4! = 24$

$\therefore$ Total No of words = 192

e $x^8 - x \equiv \left(x^2-1\right)Q(x)+ax+b$

$x=1\ \therefore\ 0=a+b$

$x=-1\ \therefore$ Remainder $R(x)=-x+1$

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QUESTION 2

a **i** $\mathrm{I} = \left[-\frac{3}{x} + \frac{5x^3}{3} \right]_1^2$

$= 13 \cdot 17$

ii $\mathrm{I} = \left[\frac{x^2}{2} + \frac{1}{2} e^{2x} \right]_0^1$

$= \frac{1}{2} e^2 \quad = 3 \cdot 69$

b $\mathrm{I} = \frac{4^{-3}}{2} \times [1 + 2\times5 + 2\times6 + 2\times8 + 9]$

$= 24 \cdot 0$

c $y = xe^x \ \therefore \ \frac{dy}{dx} = xe^x + x^x$

$\therefore \int_0^2 xe^x + e^x \, dx = xe^x \Big|_0^2 \qquad [\ fe^x \, dx = e^x$

$\therefore \int_0^2 xe^x \, dx = xe^x - e^x \Big|_0^2 = e^2 + 1 \approx 8 \cdot 4 \ (1\,\mathrm{dp})$

QUESTION 3

a **i** $20000° e^{-k.1000} = 1000°$

so $k = \frac{1}{1000} \ln 0 \cdot 05$

$= 3 \cdot 00 \times 10^3$

ii From $200 = 20000 e^{-kx}$ we get $x = \frac{\ln 100}{k} \approx 1537 \cdot 2436$

Thus, a solid surface after 1537 million years

b $x^2 + (k+2)x + 4 = 0$ has

i equal roots when $\Delta = 0$ ie $(k+2)^2 - 16 = 0$

$k = 2$ or -6

ii distinct real roots, $\Delta > 0$

ie $k < -6$ or $k > 2$

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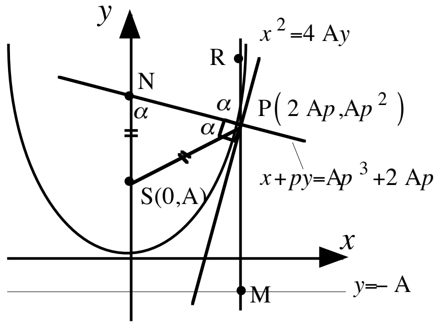
QUESTION 4

a i



Slope tan is $\dfrac{dy}{dx}=\dfrac{y'(p)}{x'(p)}$

$=\dfrac{2Ap}{2A}$

$=p$

$\therefore$ Slope $\underset{\sim}{n}$ is $-\dfrac{1}{p}$

$\therefore$ Normal is $x+py=Ap^3+2Ap$

ii Coord of focus $S\equiv(0,A)$ Coord of $\underline{N}$, $x=0$, $py=Ap^3+2Ap$

$\therefore N(0, Ap^2+2A)$

Now SP = PM (locus property of parabola) $\therefore$ SP = SN $(=Ap^2+A)$

iii Let $\angle SNP=\alpha$ $\therefore$ $\angle SPN=\angle SNP=\alpha$ (base $\angle$ of isos Δ)

$\angle RPN=\angle SNP=\alpha$ (alt $\angle$'s and NS//RP)

$\therefore$ SN and line through P // axis both make angle α with normal.

b Step 1 $n=1$, $3^1+7^2=52$ is divisible by 4

Step 2 Assume true $n=k$ Assume $3^k+7^{k+1}=4\times m$ m integer

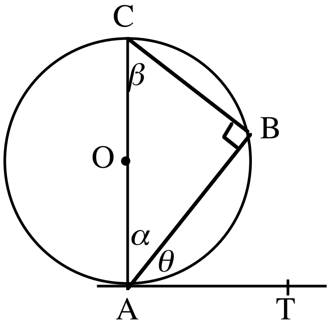
ie $7^{k+1}=4m-3^k$

Step 3 Prove true $n=k+1$ Prove $3^{k+1}+7^{k+2}=4p$, p integer

Proof : LHS $=7(4m-3^k)+3^{k+1}$ etc

QUESTION 5

a



Consider circle, centre O, with any chord AB and TA a tangent at point A.

Construction : from A draw diameter AC and join BC.

Aim : to prove $\angle BAT=\angle ACB$

Proof : Let $\angle BAT=\theta, \angle CAB=\alpha$, $\angle ACB=\beta$

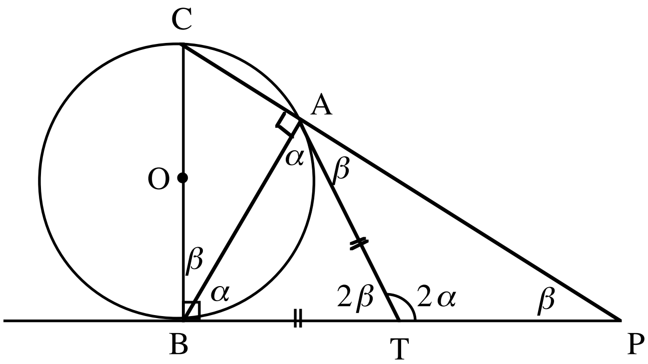
$\therefore$ $\theta+\alpha=90°$ (tang $\perp$ rad at pt of cl)

$\alpha+\beta+90°=180°$ ($\angle B=90°$ angle in ⌓; $\Delta\angle\Sigma$)

$\therefore$ $\beta+\alpha=90°$

$\therefore$ $\beta=\theta$ (both $=90-\alpha$); ie $\angle BAT=\angle ACB$

b

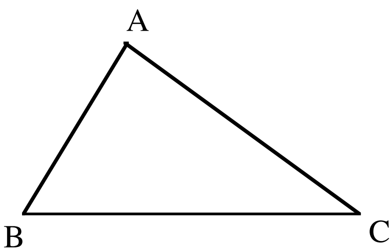


Construction : Join AB
$\therefore \angle CAB = 90°$ (Angle in semi ⊙)
$\angle CBP = 90°$ tan⊥rad
Let $\angle ABP = \alpha$
$\angle CBA = \beta$
$\therefore \alpha + \beta = 90°$

Proof : $\angle BAT = \angle ABT = \alpha$ (TA = TB, tangents to ⊙, $\therefore$ base ∠ isos Δ)
$\therefore \angle ATP = 2\alpha$ (ext ∠ of Δ = Σ int opp ∠'s)
$\angle BAP = 90°$ (Compliment to $\angle CAB = 90°$)
$\therefore \alpha + \angle TAP = 90°$ but $\alpha + \beta = 90°$
$\therefore \angle TAP = \beta$
$\therefore$ Angles of ΔBAP are $\alpha + \alpha + \beta + \angle CPB = 180°$
$\therefore \angle CPB = \beta$
$\therefore \angle ATB = 2\beta$ (ext ∠ of = Σ int opp)
ie $\angle ATB = 2\angle CPB$ Q.E.D.

QUESTION 6

a



$A + B + C = 180°$
$\therefore C = 180 - (A + B)$
$\therefore \sin C = \sin[180 - (A + B)]$
$= \sin(A + B)$
Note $\sin\theta = \sin(180 - \theta)$

$C = 180 - (A + B)$
$\therefore \tan C = \tan[180 - (A + B)]$
$= -\tan(A + B)$
$[\tan(180 - \theta) = -\tan\theta]$
ie $\tan(A + B) = -\tan C$

i $\sin C = \sin(A + B)$
$= \sin A\cos B + \cos A\sin B$

ii $\dfrac{\tan A + \tan B}{1 - \tan A\tan B} = -\tan C$
$\therefore \tan A + \tan B = -\tan C + \tan A\tan B\tan C$

ie $\tan A + \tan B + \tan C = \tan A\tan B\tan C$

b **i** $\frac{d}{dx}\left[x^4 - \ln\left(x^4+1\right)\right] = 4x^3 - \frac{4x^3}{x^4+1}$

$= \frac{4x^4 + 4x^7 - 4x^4}{\left(x^4+1\right)}$

$= \frac{4x^7}{x^4+1}$

ii $\therefore \int_0^2 \frac{x^7 dx}{x^4+1} = \frac{1}{4}\left[x^4 - \ln\left(x^4+1\right)\right]\Big|_0^2$

$\approx 3 \cdot 29$

QUESTION 7

a $\int_2^{10} \frac{x}{\sqrt{x-1}}.dx = \int_1^3 \frac{\left(t^2+1\right)2t dt}{t}$

$x = t^2 + 1$
$dx = 2t dt$
$x = 2,\ t = 1,\ x = 10,\ t = 3$

$= \frac{2t^3}{3} + 2t\Big|_1^3$

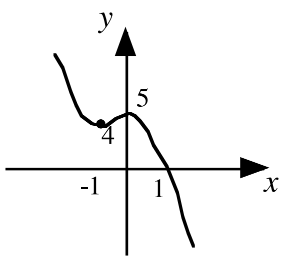
$= 21\frac{1}{3}$

b **i** $f'(x) = -6x(1+x) = 0$ at (0,5) MAX and (-1,4) MIN

ii $f''(x) = -6(1+2x)$
Now $f''(-1) = 6$ POS $f''(0) = -6$ NEG so there is change of concavity

iii $f''(x) = 0$ when $x = -\frac{1}{2}$, $\therefore$ Pt of INFLEXION at $\left(-\frac{1}{2}, 4\frac{1}{2}\right)$

iv



v We see that there is only one value of x for which $f(x) = 0$ on the real number plane.
By trial we find $f(1) = 0$

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