



Master Coaching
Ph:1800 TUITION
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Year 12 2 Unit Extension 1
Test #8

MASTER COACHING

Trial Examination

MATHEMATICS

Ext 1

Time allowed : two hours (*Plus 5 minutes reading time*)

Directions to candidates.

- * Attempt ALL questions.
- * ALL questions are of equal value.
- * All necessary working should be shown in every question.
Marks will be deducted for careless or badly arranged work.
- * A standard integral sheet will be provided.
- * Board approved calculators may be used.
- * Each question attempted is to be returned on a separate sheet of paper clearly marked question 1, question 2, etc. at the top of the page.



QUESTION 1 (Use a separate page)

- a Solve $\frac{8}{x+1} \geq x-1$ and plot the solution on a number line.
- b The point P(-3,5) divides the interval joining A(7, 19) to B(x, y) internally in the ratio of 3 : 5. Find the co-ordinates of B
- c The acute angle between the lines $y=mx$ (where $m > 0$) and $y=\frac{1}{3}x$ is 45° . Find the exact value of m.
- d How many different four letter words can be made from the word SYDNEY ?
- e Find the remainder when $x^8 - x$ is divided by $x^2 - 1$

QUESTION 2 (Use a separate page)

- a Evaluate the following integrals correct to 2 decimal places :

i $\int_1^2 \left(\frac{3}{x^2} + 5x^2 \right) . dx$ ii $\int_0^1 (x + e^{2x}) . dx$

- b The table below gives the values of $f(t)$ for $3 \leq t \leq 7$

t	3	4	5	6	7
$f(t)$	1	5	6	8	9

Use the Trapezoidal Rule with 5 function values to evaluate : $\int_3^7 f(t) . dt$ correct to 1 decimal place

- c Differentiate xe^x and hence or otherwise evaluate $\int_0^2 xe^x . dx$ correct to one decimal place.

QUESTION 3 (Use a separate page)

- a A new planet is forming as a huge mass of molten rock cools and becomes solid at the surface. The temperature T degrees at the surface after x million years is given by $T = T_0 e^{-kx}$
- i The initial temperature is 20000 degrees and after a billion years the temperature is 1000° . Find the value of k , correct to three significant figures.
- ii How old is the planet when a solid surface forms , say , at a temperature of 200° ?
- b Consider the equation $x^2 + (k+2)x + 4 = 0$
For what value of k does the equation have : i equal roots
ii distinct real roots ?

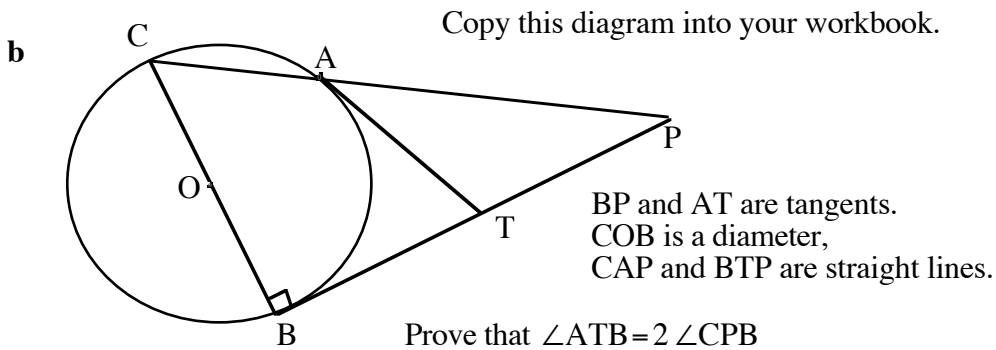


QUESTION 4 (Use a separate page)

- a i Derive the equation of the normal to the parabola $x^2 = 4Ay$ at $P(2Ap, Ap^2)$
- ii This normal meets the axis of the parabola at N and S is the focus of the parabola. Write down the co-ordinates of S and N and hence prove that $SN = SP$.
- iii Hence prove that SP and the line through P parallel to the axis of the parabola are equally inclined to the normal at P .
- b Prove by mathematical induction that $3^n + 7^{n+1}$ is divisible by 4 for all values of n .

QUESTION 5 (Use a separate page)

- a Prove that the angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment.



QUESTION 6 (Use a separate page)

- a If $A + B + C = 180^\circ$ explain why $\sin C = \sin(A + B)$ and that $\tan(A + B) = -\tan C$
 Hence prove that in any triangle that : i $\sin C = \sin A \cos B + \cos A \sin B$
 ii $\tan A + \tan B + \tan C = \tan A \tan B \tan C$
- b i Differentiate with respect to x : $x^4 - \ln(x^4 + 1)$, simplifying the result to one term.



ii Hence evaluate $\int_0^2 \frac{x^7}{x^4 + 1} dx$ correct to two decimal places.

QUESTION 7 (*Use a separate page*)

a Evaluate $\int_2^{10} \frac{x}{\sqrt{x-1}} dx$ using the substitution $x=t^2 + 1$

b Consider $f(x)=5-3x^2 -2x^3$

i Find the co-ordinates of the stationary points on the graph $y=f(x)$

ii Establish the nature of these points.

iii Locate the point of inflexion.

iv Sketch the graph.

v Explain how the graph shows that the equation $5-3x^2 -2x^3=0$ has just one real root.
Find the real root.



Standard Integrals $\int x^n \cdot dx = \frac{1}{n+1} x^{n+1} + c$; $n \neq -1$, $x \neq 0$ if $n < 0$

$$\int \frac{1}{x} \cdot dx = \ln x + c \quad ; \quad x < 0$$

$$\int e^x \cdot dx = e^x + c$$

$$\int \cos x \cdot dx = \sin x + c$$

$$\int \sin x \cdot dx = -\cos x + c$$

$$\int \sec^2 x \cdot dx = \tan x + c$$

$$\int \sec x \cdot \tan x \cdot dx = \sec x + c$$

$$\int \frac{1}{1+x^2} \cdot dx = \tan^{-1} x + c$$

$$\int \frac{1}{\sqrt{1-x^2}} \cdot dx = \sin^{-1} x + c$$