



Master Coaching
Ph:1800 TUTOR
www.mastercoaching.com

Year 12 2 Unit Extension 1
Test #9

Trial HSC

MATHEMATICS.

Extension 1

Time allowed : two hours (*Plus 5 minutes reading time*)

Directions to candidates.

- * Attempt ALL questions.
- * ALL questions are of equal value.
- * All necessary working should be shown in every question.
Marks will be deducted for careless or badly arranged work.
- * A standard integral sheet will be provided.
- * Board approved calculators may be used.
- * Each question attempted is to be returned on a separate sheet of paper
clearly marked question 1, question 2, etc. at the top of the page.

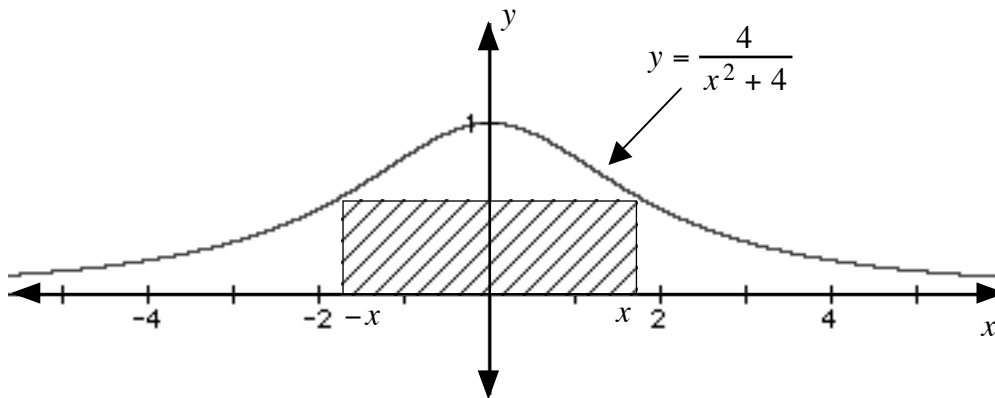


Question 1 *Start a new page.*

- a Differentiate : **i** $\log_e(x^2 + 2x)$ **ii** $e^{x^2} \cdot \cos(x^2)$
- b Describe and sketch the region $1 > y > |x - 1|$
- c Evaluate **i** $\int_0^2 x e^{x^2} \cdot dx$ **ii** $\int_0^1 \frac{x+2}{\sqrt{1-x^2}} dx$
- d A person tosses two fair coins and upon examination claims 'one of the coins is a head'.
Find the probability that both coins are heads.
A second person tosses two fair coins - a 20c and a 10c coin.
If he claims that the 10c coin shows a head find the probability that both coins are heads.

Question 2 *Start a new page.*

- a A rectangle is inscribed under the curve $y = \frac{4}{x^2 + 4}$ as shown below,
such that the rectangle is symmetrical about the y - axis.
Prove that the maximum area of this rectangle is two square units.



- b If $A_{\max} = 2 \int_0^2 \frac{4}{x^2 + 4} \cdot dx$,

use Simpsons Rule with two strips to evaluate A_{\max} correct to 4 decimal places.

- c Find the exact value of A_{\max}
Compare this with the approximate answer given by Simpson's Rule ... Comment.



Question 3 *Start a new page.*

a Prove by induction that $1 \times 5 + 2 \times 5^2 + 3 \times 5^3 + \dots + n \times 5^n = \frac{5 + (4n-1) \times 5^{n+1}}{16}$

b If $f(x) = x - \ln(x+1)$, show that $\frac{x}{2} < f'(x) < x$ for $0 < x < 1$

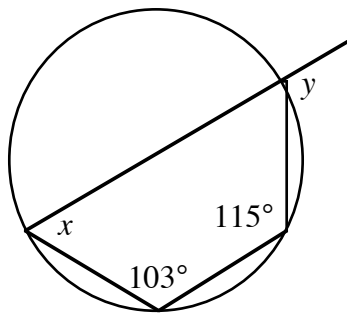
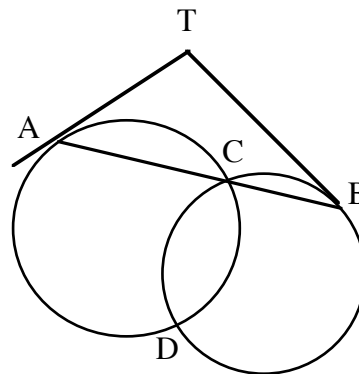
Hence, by integrating, show that $\frac{x^2}{4} < x - \ln(x+1) < \frac{x^2}{2}$

and deduce that $4 < \int_0^1 \ln(x+1)^2 \cdot dx < 5$

Question 4 *Start a new page.*

a Complete the statement :
 “The angle between a tangent to a circle and a chord drawn to the point of contact ...”

b i In the figure, ACB is a straight line,
 TA & TB are tangents to the circles.
 Prove that TADB is a cyclic quadrilateral.



ii In the diagram evaluate x & y

c i Find the remainder when $x^{10} + 7x^2 - 3x + 5$ is divided by $x^2 - 1$

ii If the roots of $4x^3 - 8x^2 + 6x - 7 = 0$ are α , β and γ , find the values of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

