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Year 12 2 Unit Extension 1

Test #10 Answers

QUESTION 1

a **i** $\cos(A+B)=\cos A\cos B-\sin A\sin B$ Put $A=B=x$

$\therefore\ \cos(2x)=\cos^2 x-\sin^2 x;\ \cos^2 x=1-\sin^2 x$

$\cos 2x=1-2\sin^2 x$

$\therefore\ \sin^2 x=\dfrac{1-\cos 2x}{2}$

ii $\therefore\ \int_0^1 6\sin^2 4\theta.d\theta=\int_0^1 3-3\cos 8\theta d\theta$

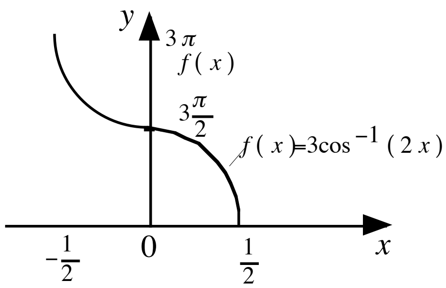
$=3\theta-\frac{3}{8}\sin 8\theta\Big|_0^1$

$\approx 2\cdot 629\quad (3\text{dp})$

b $f(x)=3\cos^{-1}(2x)$ Domain $-1\le 2x\le 1$

ie $-\frac{1}{2}\le x\le\frac{1}{2}$

Range $0\le\dfrac{f(x)}{3}\le\pi$ ie Range $0\le f(x)\le 3\pi$



c **i** No of Teams $={}^9C_6\times{}^{10}C_7=10080$

ii No of Teams with both brothers $={}^9C_6\times{}^8C_5=4704$

$\therefore$ No of Teams where at least 1 brother misses $=10080-4704$

$=5376$

$\therefore$ Pr (at least 1 brother misses selection) $=\dfrac{5376}{10080}$

$=\dfrac{8}{15}$

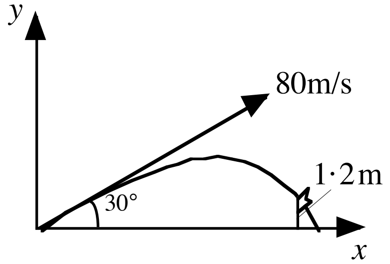
QUESTION 2

a $\left(3x^2+\frac{2}{x}\right)^{11}$ $\quad T_{k+1}={}^{11}C_k\left(3x^2\right)^{11-k}\left(\frac{2}{x}\right)^k$

Power of $x=22-2k-k=1\quad\therefore\ k=7$

$\therefore$ Coefficient of $x={}^{11}C_7\times 3^4\times 2^7$

b i



$\ddot{y} = -10$

$\dot{y} = -10t + c$

when $t = 0,\ y = 40\text{m/s}\ \therefore\ C_1 = 40\text{m/s}$

$\therefore\ \dot{y} = -10t + 40$

$\therefore\ y = -5t^2 + 40t + C_2,\ y = 0,\ t = 0,\ C_2 = 8$

$\ddot{x} = 0$

$\dot{x} = 40\sqrt{3}$

$x = 40\sqrt{3}t + C_3 \qquad t = 0,\ x = 0\ \therefore\ C_3 = 0$

ii $y = 1\cdot2$ when $-5t^2 + 40t = 1\cdot2$

ie $t^2 - 8t + 0\cdot24 = 0$

$\therefore\ t = \dfrac{8 + \sqrt{64 - 0\cdot96}}{2}$ on down path

$\approx 7\cdot97$

$\therefore$ Distance to pin $= 552\text{m}$

$= 550\text{m}$ (nearest 10m)

c $y = x^2 + x + 7 \cap y = 2x^2 - 2x - 3$ when

$2x^2 - 2x - 3 = x^2 + x + 7$

$x^2 - 3x - 10 = 0$

$x = -2$ or 5

$$\text{Area} = \int_{-2}^{5} -x^2 + 3x + 10\,dx$$

$$= -\frac{1}{3}x^3 + \frac{3}{2}x^2 + 10x \Big|_{-2}^{5}$$

$$= 57\frac{1}{6}$$

QUESTION 3

a i Given $v^2 = -25x^2 + 150x - 200$

then $\ddot{x} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = -25x + 75 = -25(x - 3)$

$\therefore$ Acceleration 2 displacement and directed towards the centre of motion

$\therefore$ motion is S.H.M.

$\ddot{x} = -25(x-3)$ of form $\ddot{X} = -n^2 X$

ii Centre of the motion $x = 3\,\text{m}$

iii Period of the motion $x = \dfrac{2\pi}{n} = \dfrac{2\pi}{5}\,\text{sec}$

iv MAX/MIN displacement occurs when $v^2 = 0$

ie $-25x^2 + 150x - 200 = 0$

$x^2 - 6x + 8 = 0$ $\quad x = 2$ or 4

$\therefore$ Amplitude = 1m

v Maximum speed occurs when $x = 3$

MAX Speed = 5m/s^2

b $f(x) = x + \ln x$; $\quad f'(x) = 1 + \dfrac{1}{x}$; $\quad f'(0\cdot5) = 3$

$x_1 = 0\cdot5$, $\quad x_2 = 0\cdot5 - \dfrac{f(0\cdot5)}{f'(0\cdot5)}$ $\qquad f(0\cdot5) = -0\cdot193$

$= 0\cdot5 + \dfrac{0\cdot193}{3} \approx 0\cdot56 \;(2\,\text{dp})$

QUESTION 4

a Let x be number of Grubers caught/wk

	80% Chance			
	$\Pr(x=0) \to \Pr(x=4)$	$\Pr(x=5)$	$\Pr(x=6)$	$\Pr(x=7)$
	$0\cdot148$	$0\cdot275$	$0\cdot367$	$0\cdot210$
Expected Return	-$7400	$13750	$22020	$14700

i Cost of buying & operating 7 traps for 1 year $= \$(7 \times 3500 \times 52) = \$1274000$

ii Return from selling Grubers $= \{(\$13750 + \$22020 + \$14700) - \$7400\} \times 52$

$= \$2239640$

$\therefore$ Net Gain/Yr $= \$965640$

iii

	90% Chance			
Probs	$0\cdot026$	$0\cdot124$	$0\cdot372$	$0\cdot478$
Expected Return	\$1300	\$6200	\$22300	\$33460

Cost of operating improved traps $= 2\times\$127400$
$= \$2548000$

Return from sales $= \$(6200+22300+33460-1300)\times 52$
$= \$3154320$

$\therefore$ Profit/Yr $= \$3154320 - \$2548000$
$= \$606320$

b $\displaystyle\int_3^6 \frac{3x}{\sqrt{x-2}}.dx = 6\int_1^2 (u^2+2)\,du$
$= 26$

QUESTION 5

a $\sin\theta + 2\cos\theta = 1\cdot5 = A\sin(\theta+\alpha)$

$\therefore A\cos\alpha = 1 \qquad A=\sqrt{5} \qquad \tan\alpha = 2,\ \alpha = 1\cdot107\ (3\text{dp})$

$A\sin\alpha = 2$

$\therefore \sqrt{5}\sin(\theta+1\cdot107) = 1\cdot5$

$\therefore \theta + 1\cdot107 = 0\cdot735 \text{ or } 2\cdot406 \text{ or } 7\cdot018$

$\therefore \theta = 1\cdot299 \text{ or } 5\cdot911$

b i $T = A + Ce^{kt} \quad -(1) \qquad \therefore Ce^{kt} = T - A$

$\dfrac{dT}{dt} = k\cdot Ce^{kt} \quad -(2) \qquad \therefore (1)$ is a solution of (2)

$= k(T-A)$

ii $t=0 \quad T=2 \qquad \therefore 2 = A + C;\quad A = 30$

$\therefore C = -28$

$\therefore T = 30 - 28e^{kt}$

$t=30,\ \ \mathrm{T}=10\ \ \therefore\ \ 10=30-28e^{30k}$

$e^{30k}=\frac{20}{28},\ k\approx-0\cdot0112$

$t=75$ then $\mathrm{T}=30-28e^{75k}$

$=18^\circ\mathrm{C}$

iii As $t\to\infty$ $\quad \mathrm{T}\to30^\circ\mathrm{C}$ the air temperature

c $\ {}^{n-1}C_{r-1}+{}^{n-1}C_r=\frac{(n-1)!}{(n-k)!(k-1)!}\times\frac{k}{n}+\frac{(n-1)!}{(n-k-1)!\times k!}\times\frac{(n-k)}{(n-k)}$

$=\frac{(n-1)![k+n-k]}{(n-k)!k!}$

$=\frac{n!}{(n-k)!k!}$

$={}^{n}\mathrm{C}_{k}$

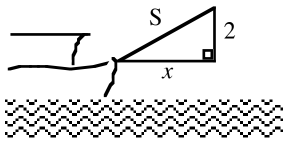
QUESTION 6

a Let amt/mth $=\$\mathrm{P}$ $\quad\therefore\quad \frac{\$\mathrm{P}(1\cdot005)\left[(1\cdot005)^{48}-1\right]}{1\cdot005-1}=\$20000$

$\therefore\quad \$\mathrm{P}=\frac{\$2000\times0\cdot005}{1\cdot005\left[(1\cdot005)^{48}-1\right]}$

$\approx\$367\cdot86$

b



$S=\sqrt{x^2+4}$ $\quad \frac{dS}{dt}=1\cdot 5\,\text{m/s}$

$\frac{dS}{dx}=\frac{x}{\sqrt{x^2+4}}$

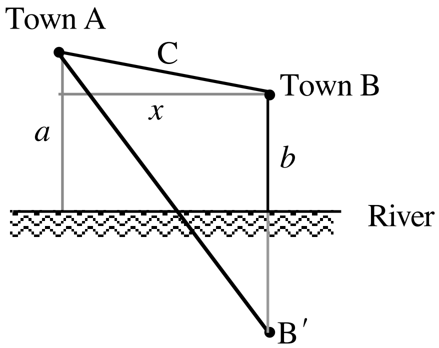
$\therefore \frac{dx}{dt}=\frac{dx}{dS}\times\frac{dS}{dt}\Big|_{x=\sqrt{21}}$

$=\frac{5}{\sqrt{21}}\times 1\cdot 5\,\text{m/s}$

$=\frac{7\cdot 5}{\sqrt{21}}\,\text{m/s}$

QUESTION 7

a



Least length of pipe $= AB'$

$x^2=C^2-\left(a-b^2\right)$

$\therefore\ AB'=\left(a+b^2+x^2\right) =\left(a+b^2\right)+c^2-\left(a-b^2\right)$

$=c^2+4ab$

$\therefore$ Least length of pipe $=\sqrt{C^2+4ab}$

b **i** No of way $={}^5C_3\times{}^4C_2\times 5!=7200$

ii Pr (novels read after each other) $=\frac{4!\times 2!}{5!}=\frac{2}{5}$

c $x^{10}-10=(x-1)(x+1)Q(x)+ax+b$; Remainder $=-9x$