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Year 12 2 Unit Extension 1

Test #10

MATHEMATICS

Ext 1

Time allowed : two hours *(Plus 5 minutes reading time)*

Directions to candidates.

* Attempt ALL questions.
* ALL questions are of equal value.
* All necessary working should be shown in every question.
  Marks will be deducted for careless or badly arranged work.
* A standard integral sheet will be provided.
* Board approved calculators may be used.
* Each question attempted is to be returned on a separate sheet of paper clearly marked question 1, question 2, etc. at the top of the page.

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QUESTION 1 *(Start a new page)*

a **i** Given that $\cos(A+B)=\cos A.\cos B-\sin A.\sin B$, prove that $\sin^2 x=\frac{1-\cos 2x}{2}$

ii Hence or otherwise find $\int_0^1 6\sin^2 4\theta.d\theta$ correct to three decimal places.

b Write down the domain and range of the function $f(x)$ where $f(x)=3\cos^{-1}(2x)$
Hence sketch the curve.

c A team of 13 players which must consist of 6 forwards and 7 backs is to be selected from a squad of 9 forwards and 10 backs.

i How many such teams can there be formed ?

ii Two brothers, both backs, try out for the team.
Find the probability that at least one of the brothers misses selection ?

QUESTION 2 *(Start a new page)*

a Find the coefficient of x in the expansion of $\left(3x^2+\frac{2}{x}\right)^{11}$

b A golfer hits her ball in such a way that it has an initial velocity of 80m/s and the ball is projected at an angle of $30°$ to the horizontal.
The golf ball hits the flag pin at a height of $1{\cdot}2$m when the ball is decending.

i Write down and solve the equations of motion (use g = -10m/s2)

ii How far is the golfer from the pin when she hits the ball ? (answer to the nearest 10 m.)

c Find the area between the curves $y=x^2+x+7$ and $y=2x^2-2x-3$

QUESTION 3 *(Start a new page)*

a A particle moves in such a way that its displacement x metres from a given origin $x=0$ at time t sec is given by the equation $v^2=-25x^2+150x-200$
where v is the velocity in metres per second.

i Prove that the motion is simple harmonic.

Find the : **ii** Centre of the motion
iii Period of the motion
iv Amplitude of the motion
v Maximum speed of the particle and the value x of when the maximum speed occurs.

b Taking $x=0{\cdot}5$ as the first approximation to the root of $x+\ln x=0$,
use Newton's Method to find the second approximation. (answer correct to 2 decimal places)

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QUESTION 4 *(Start a new page)*

a Brenton has won a contract to supply the Japanese with an exotic fish called a 'gruber' which can only be caught in Australian waters. The most prolific fishing ground is known only to Brenton. The terms of the contract are these:

#1 : Japan will pay \$10000 for each gruber supplied fresh
(grubers cannot be stored for more than 1 week)

#2 : If Brenton fails to supply at least 5 grubers in any one week there is a penalty of \$50000 for him to pay and he receives nothing for the grubers supplied for that week.

The cost of catching grubers has been calculated to be :

Each gruber trap costs \$100000. The average life span of a gruber trap is one year.
Each trap cost \$1000 per week to operate.
The money to buy the traps has to be borrowed and repaid at a rate of \$2500 /week per trap.

There are no other expenses.

Brenton has noticed over a long period of time that each gruber trap has an 80% chance of catching a gruber in any one week.

If Brenton can only operate 7 traps in a one week period,

find : **i** The total cost of buying and operating the 7 traps over a period of 52 consecutive weeks.

ii The expected profit (or loss) that Brenton will make in any one week and hence Brenton's total profit or loss for the year.

iii Suppose Zak bought an improved trap at twice the cost overall, but which had a 90% chance of catching a gruber in any one week. What then would then be Brenton's expected profit or loss for the year ?

b Evaluate $\int_3^6 \frac{3x}{\sqrt{x-2}}.dx$ using the substitution $u^2 = x-2$

QUESTION 5 *(Start a new page)*

a Find all the angles θ with $0 \le \theta \le 2\pi$ for which $\sin\theta + 2\cos\theta = 1{\cdot}5$

b Assume that the rate at which a mass warms in air is proportional to the difference between its temperature T and the constant temperature A of the surrounding air. This rate can be expressed by the differential equation $\frac{dT}{dt} = -k(T - A)$ where t is the time in minutes and k is a constant.

i Show that $T = A + Ce^{kt}$, where C is a constant, is a solution of the differential equation.

ii A body warms from 2°C to 10° C in 30 minutes. Air temperature around the body is 30° C . Find the temperature of the body after a further 45 minutes have elapsed. (to the nearest degree)

iii By referring to the equation for T, explain the behavior of T as t becomes large.

c Given that ${}^nC_r = \frac{n!}{r!(n-r)!}$ prove the Pascal Triangle relationship ie ${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$

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QUESTION 6 *(Start a new page)*

a When Jane began work at age 17 years she started saving for a trip to be taken in four years time. She will need \$20000 to pay for the trip.
Jane commits herself to pay the same amount \$P at the start of each month , starting immediately. She will take out her investment at the end of the 48th month.

If Jane's investment earns for her $0{\cdot}5\%$ per month and interest is compounded monthly, find the value of \$P which will earn Jane a return of \$20000.

b A boat is pulled into a dock by means of a rope with one end attached to the bow of the boat, the other end going through a ring attached to the dock at a point 2m higher than the bow of the boat.

If the rope is pulled in at the rate of $1{\cdot}5$m/s, how fast is the boat approaching the dock when 5m of rope are out ?

QUESTION 7 *(Start a new page)*

a Two towns located on the same side of a straight river, agree to construct a pumping station and filtering plant at the water's edge, to be used jointly to supply the towns with water. The distance of the two towns from the river are "a" and "b" . The distance between them is "c"

Show that the sum of the lengths of the pipes joining them to the pumping station is at least as great as $\sqrt{c^2+4ab}$

b From 5 novels and 4 plays, a student must select 3 novels and 2 plays.

i In how many orders may the 5 texts be selected and read.

ii Find the probability that the 2 plays are read one immediately after the other.

c Find the remainder when $x^{10}-10$ is divided by x^2-1

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